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High-Field Magnetotransport in a Percolating Medium.

A. K. SARYCHEV (*) (**), D. J. BERGMAN (**) and Y. M. STRELNIKER (**)

(*) *Institute for High Temperature of the Russian Academy of Sciences Moscow, Russia* (§)

(**) *School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Sciences, Tel Aviv University - Tel Aviv 69978, Israel*

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Abstract. – A discrete network model is used to discuss the critical behaviour of magnetotransport in a percolating medium in the presence of a magnetic field H of arbitrary strength. By applying a real-space renormalization group transformation, we find that there is strong magnetoresistance $\rho(H)$ near the percolation threshold. We also find that there are two fixed points: one at $p = p_c$ and $H = 0$, and another at $p = p_c$ and $H = \infty$. The crossover between them is governed by a new, field-dependent length. In a percolating metal-insulator mixture, the resistivity ratio with and without a field $\rho(H)/\rho(0)$ is predicted to saturate as $p \to p_c$ at a value $\sim H^{0.5}$.

In spite of the considerable effort devoted by theorists during the last 15 years to the study of percolating systems, relatively little work has been done on electrical conduction in the presence of a magnetic field, and almost all of that was restricted to a weak magnetic field [1-9]. Even those few studies resulted in some novel predictions, later verified by experiments, such as the fact that either component in a metal–non-metal composite could dominate the critical behaviour of the weak-field Hall effect, both above and below the percolation threshold of the metallic component [6, 10]. The only existing theoretical studies of high-field magnetotransport in three-dimensional (3D) composite media are based on an effective medium approximation (EMA) [11-14]. From this analysis it appeared that resistivity at a percolation threshold is insensitive to the presence of a magnetic field. A similar result can also be obtained from the links-nodes (LN) model for the backbone of the infinite cluster. Nevertheless, some simple considerations indicate that the real situation may actually be quite different. Consider the sample configurations of 3D percolation channels shown in fig. 1. Suppose that the conducting component has no intrinsic magnetoresistance at all, *i.e.* its ohmic resistivity r_0 does not depend on the magnetic field. Then the thin, singly connected channel of fig. 1*a*) also has no magnetoresistance. The situation is quite different in the case of the channel with a transverse loop of fig. 1*b*). In the

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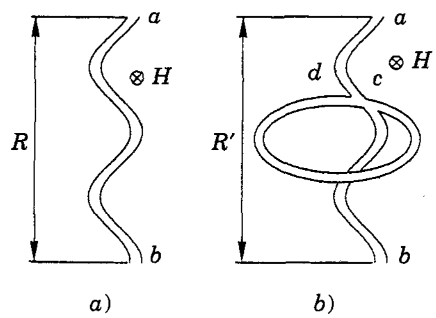

Fig. 1. – *a*) Singly connected percolation channel with resistance R. *b*) Percolation channel with a single loop *cd*. The resistance $R'(H)$ depends on the magnetic field applied in the plane of the loop.

absence of a magnetic field, the loop *cd* has almost no effect upon the resistance $R'(0)$ between *a* and *b* ($R'(0) \simeq R$). But when a strong magnetic field H is present, the Hall resistivity r_{H0} can be much larger than r_0 since $r_{H0}/r_0 \sim H$. A unit current flowing in the channel *ab* then produces a large Hall e.m.f. $\sim$ H in the loop *cd*. This produces an Ohmic current $\sim H$ in the loop, and that, in turn, produces an additional Hall voltage $\sim H^2$ in the *ab* channel. This effect leads to a large magnetoresistance which is quadratic in r_{H0}: $R'(H)/R \sim H^2$. Our tentative conclusion from this qualitative analysis is that the microstructural details of the percolating cluster may have important and unexpected effects on the magnetoresistance at high fields. Therefore one ought to investigate this problem using other means than EMA or LN. We note here that in 2D the duality transformation has been used to prove some exact theorems [15, 16] which enabled the critical behaviour near a percolation threshold p_c to be worked out exactly for arbitrary field strengths [17, 18].

Here we report on results of some calculations using a discrete network model for magnetotransport in a random percolation medium at strong magnetic fields. This model was used earlier for discussing the Hall effect in percolating systems at *low fields* only [9]. In the presence of a magnetic field the discretization is a non-trivial procedure, since at least one seemingly obvious model turned out to be in a different universality class of critical properties from that of real, percolating continuum composites [4, 7]. The 2D version of the model is first shown to undergo a simple duality transformation which ensures that it has the correct critical behaviour at any field strength. We then describe results of a real-space renormalization group (RSRG) transformation which we applied to the 3D version of the model. It is shown that there is a strong magnetoresistance near the percolation threshold. On the basis of these results we propose a new type of scaling behaviour at high fields. This behaviour is dominated by two fixed points, at $H = 0$ and at $H = \infty$, with a crossover between them at an intermediate value of H which depends on p(or on the linear size L in the case of a system exactly at p_c).

The model is constructed from discrete network elements, each of which has six terminals, one along each ray of the coordinate axes (see fig. 2*a*)), at which potentials are applied and electric currents flow. The electrical properties of the element are characterized by a 5×5 resistance matrix $\widehat{R}$ which relates the voltages $V_i - V_1$ to the current $I_j - I_1$, $i, j = 2, \ldots, 6$ (I_1 is not independent of the other I_j, since $\sum_1^6 I_j = 0$)

$$V_i - V_1 = \sum_{j=2}^{6} R_{ij} I_j \quad \text{for } i = 2, \ldots, 6. \tag{1}$$

In the presence of a magnetic field in the x-direction this matrix has both a symmetric and an

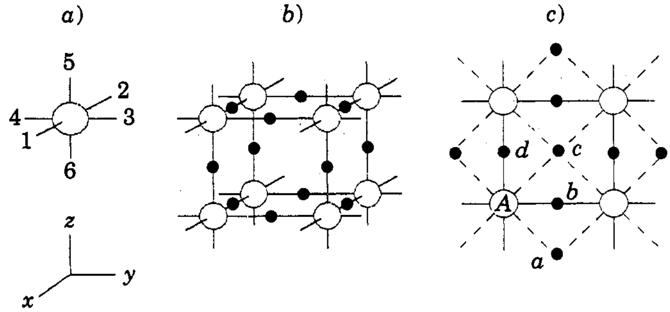

Fig. 2. – *a*) The basic discrete circuit element which mimics the magnetotransport properties of a real material. The numbering of the terminals corresponds to the rows and columns of the resistance matrix $\widehat{R}$. *b*) One unit cell of a simple cubic network of the basic elements. Each element is marked by an empty circle. The filled circles mark the connection points between terminals of adjacent elements. *c*) A portion of the square network made of 4-terminal 2D elements like *a*) (full lines), along with the dual network (dashed lines). The empty circles again mark the elements, while the black circles mark connections between terminals of different elements. The current flowing to the right at the connection point *b* of the original network is the same as the voltage $V_c - V_a$ between the connection points *a*, *c* of the dual network. Similarly, the voltage $V_d - V_b$ is the same as the current flowing in the dual network out of the element *A* through the connection point *c*.

antisymmetric part, and is given by

$$\widehat{R} \equiv r_0 \begin{bmatrix} 2 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 & 1 \\ 1 & 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 & 2 \end{bmatrix} + r_{H0} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \end{bmatrix}. \tag{2}$$

This form represents a material with an Ohmic resistivity $2r_0$, a Hall resistivity $2r_{H0}$ and zero magnetoresistance. These elements are placed randomly at a fraction p of the sites of a simple cubic lattice, the other sites remaining empty. Adjacent terminals from neighbouring elements are connected to each other at the centre of the appropriate network bond (see fig. 2*b*)).

The 2D version of this model is obviously a simple square lattice of 4-terminal elements described by a 3×3 matrix that can easily be constructed by analogy with the matrix $\widehat{R}$ of (2). Somewhat less obvious, but nevertheless true, is the fact that a duality transformation exists for this model. In this transformation, each element is replaced by a similar one that has its terminals rotated by 45° in the plane, and terminals from different elements are connected at the centres of the square unit cells (see fig. 2*c*)). The resistance matrix $\widehat{R}$ must also be inverted. Voltages between adjacent terminals of the same original element become the terminal currents of the transformed element and *vice versa* (see fig. 2*c*)), so that a solution of Kirchhoff's equations for the potentials and currents of the original network is automatically transformed into a similar solution for the dual network. This is easily seen to be the analogue of the duality transformation for continuum 2D conductors that was used in ref. [15, 16]. Note that the topology of the dual network differs from that of the original network, and that they have different (in fact, complementary) percolation thresholds. The existence of this transformation ensures that the magnetotransport tensor has the same critical behaviour in the case of the 2D network model as in the case of a real, percolating composite medium. Models which do not have such a duality transformation are in general unable to mimic the critical behaviour of a real composite medium both in 2D and in 3D [4].

In order to study the critical behaviour of this model in 3D near p_c, we applied a RSRG transformation to the $2 \times 2 \times 2$ random network shown in fig. 2*b*) [19]: all 256 configurations were constructed, and then each of them was transformed into a single, 6-terminal element by joining together each set of four external terminals that extend in the same direction (see fig. 2*b*)). An element constructed in this way was characterized by a transformed resistance matrix $\hat{R}$, and current was allowed to flow only in the y-direction. Applying a strong field H (*i.e.* $r_{H0} \gg r_0$) in the x-direction, the resulting transverse Ohmic and Hall voltages were used to calculate the corresponding resistivities of the transformed element. The inverse Ohmic resistivities and the Hall mobility were then averaged over all the configurations that percolate in the y-direction using the weights appropriate for p_c, as calculated from this RSRG (namely, $p_c \approx 0.28184$). A similar calculation was performed with the magnetic field in the y-direction in order to obtain the longitudinal magnetoresistance. The averaged Hall mobility and magnetoresistivities were then used to iterate this RSRG n times, up to a system of total linear size $L = 2^n$. This simple procedure nevertheless permitted us to incorporate many non-trivial configurations in the percolation cluster such as singly connected chains, loops, etc. We believe that it gives a *qualitatively* correct picture of the high-field transport properties—we know it does for the low-field properties.

The results of these RSRG calculations are as follows: at first the transverse and the longitudinal resistances remain almost equal and independent of r_{H0}, scaling mainly with size but in a different way than in the absence of a magnetic field. Denoting the Ohmic and Hall resistance after n iterations by $r(n)$, $r_H(n)$, we find that

$$r(n+1) \approx \lambda_r r(n) + \lambda_1 \frac{r^3(n)}{r_H^2(n)}, \tag{3}$$

$$\frac{r_H(n+1)}{r(n+1)} \approx \lambda_H \frac{r_H(n)}{r(n)} + \lambda_2 \frac{r(n)}{r_H(n)}, \tag{4}$$

as long as $r_H(n) \gg r(n)$. This leads to a scaling behaviour for the specific resistivities $\rho \equiv rL$, $\rho_H \equiv r_H L$ of the form

$$\rho = r_0 L^{\tilde{t}_H}, \qquad \tilde{t}_H = 1 + \frac{\ln \lambda_r}{\ln 2} \approx 2.669, \tag{5}$$

$$\frac{\rho_H}{\rho} = \frac{r_{H0}}{r_0} L^{\tilde{g}_H - \tilde{t}_H}, \qquad \tilde{g}_H = \tilde{t}_H + \frac{\ln \lambda_H}{\ln 2} \approx 0.927. \tag{6}$$

Sincle clearly $\lambda_H < 1$, therefore when n is sufficiently large, the strong-field condition $r_H(n) \gg r(n)$ will be violated. When n is increased beyond that point, one eventually arrives at the opposite weak-field regime, where $r_H(n) \ll r(n)$. In that regime, the transverse and longitudinal Ohmic resistances are again almost equal, and under the RSRG transformation $r(n)$ and $r_H(n)/r(n)$ again scale almost independently as follows:

$$r(n+1) \approx \omega_r r(n) + \omega_1 \frac{r_H^2(n)}{r(n)}, \tag{7}$$

$$\frac{r_H(n+1)}{r(n+1)} \approx \omega_H \frac{r_H(n)}{r(n)} + \omega_2 \frac{r_H^3(n)}{r^3(n)}, \tag{8}$$

as long as $r_H(n) \ll r(n)$. This leads to a critical behaviour of the form

$$\rho \sim L^{\tilde{t}}, \qquad \tilde{t} = 1 + \frac{\ln \omega_r}{\ln 2} \approx 1.753, \tag{9}$$

$$\frac{\rho_H}{\rho} \sim L^{\tilde{g}-\tilde{t}}, \qquad \tilde{g} = \tilde{t} + \frac{\ln \omega_H}{\ln 2} \approx 0.721, \tag{10}$$

where the coefficients are now determined by the values of $r(n)$, $r_H(n)$ at the crossover point. This will have the effect of introducing a magnetic-field dependence into the Ohmic resistivity. Since clearly $\omega_H < 1$, the ratio $r_H(n)/r(n)$, which is already much less than 1, continues to decrease with increasing n. Note that the critical exponents are different in the two regimes, and that $\tilde{t}_H > \tilde{t}$, $\tilde{g}_H > \tilde{g}$, as might be expected on physical grounds, since only a fraction of the entire percolating backbone is expected to carry significant current in the high-field regime.

The behaviour described above is expected to hold at least *qualitatively* for the exact renormalization group transformation of this model, although the precise values of the critical exponents will of course be different. Thus, we know that the weak-field exponents $\tilde{t} \equiv t/\nu$, $\tilde{g} \equiv g/\nu$ (where ν is the percolation correlation length exponent) are actually about 2.2, 0.4, respectively [7].

What these results mean is that for the percolating metal-insulator mixture in the presence of a magnetic field H (in this article, H simply represents the ratio r_{H0}/r_0, which for a free electron metal is equal to $\omega_c\tau$, where ω_c is the cyclotron frequency and τ is the conductivity mean free time), there are two fixed points: one at $p = p_c$ and $H = 0$, stable against a perturbation to $H \neq 0$, and another at $p = p_c$ and $H = \infty$, unstable against a perturbation to $H < \infty$. When H is large but not infinite, *i.e.* the initial ratio of Hall to Ohmic resistance $r_{H0}/r_0 \gg 1$, a crossover occurs, with increasing system size $L = 2^n$, from a behaviour dominated by the fixed point at $H = \infty$ to a behaviour dominated by the fixed point at $H = 0$. As is often done in such cases, we propose to describe this crossover by a scaling ansatz, namely

$$\rho(L, H) = r_0 L^{\tilde{t}_H} F_\rho\left(\frac{L}{\xi_H}\right), \tag{11}$$

$$\rho_H(L, H) = r_{H0} L^{\tilde{g}_H} F_H\left(\frac{L}{\xi_H}\right), \tag{12}$$

where

$$\xi_H \equiv \left(\frac{r_{H0}}{r_0}\right)^{\frac{1}{\tilde{t}_H - \tilde{g}_H}}, \qquad \frac{1}{\tilde{t}_H - \tilde{g}_H} \approx 0.574 \tag{13}$$

is a new, field-dependent length which increases with increasing H (actually, with increasing r_{H0}/r_0). In order to describe correctly the critical behaviour in the strong-field limit, $F_\rho(x)$ and $F_H(x)$ must both tend to non-zero, finite, magnetic-field–independent values as $x \to 0$. In order to also describe correctly the weak-field limit, we must have, for $x \to \infty$,

$$F_\rho(x) \to \text{const} \times x^{\tilde{t} - \tilde{t}_H}, \tag{14}$$

$$F_H(x) \to \text{const} \times x^{\tilde{g} - \tilde{g}_H}, \tag{15}$$

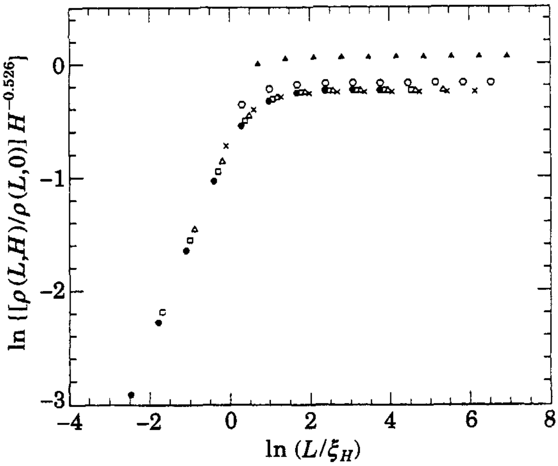

Fig. 3. – Log-log plot of the ratio $\rho(L,H)/\rho(L,0)$ *vs.* L/ξ_H. The values of L go up to 1024, while the values of H, measured in units of the Hall-to-Ohmic resistance ratio r_{H0}/r_0, go up to 256: ▲ $H=1$, ○ $H=2$, × $H=4$, △ $H=16$, □ $H=64$, ● $H=256$.

and consequently

$$\frac{\rho(L,H)}{\rho(L,0)} \to \text{const} \times \xi_H^{\tilde{t}_H-\tilde{t}} \sim \left(\frac{r_{H0}}{r_0}\right)^{\frac{\tilde{t}_H-\tilde{t}}{\tilde{t}_H-\tilde{g}_H}} \qquad \text{as } L\to\infty\,, \tag{16}$$

$$\frac{\tilde{t}_H-\tilde{t}}{\tilde{t}_H-\tilde{g}_H} \approx 0.526\,. \tag{17}$$

Using (7), we can also derive the manner in which the ratio of resistivities tends to its asymptotic, H-dependent value with increasing L,

$$\frac{\rho(L,H)}{\rho(L,0)} - \lim_{L=\infty}\frac{\rho(L,H)}{\rho(L,0)} \sim L^{-2(\tilde{t}-\tilde{g})}\,\xi_H^{\tilde{t}+\tilde{t}_H-2\tilde{g}}\,. \tag{18}$$

These expectations have been tested against our approximate RSRG transformation by plotting the ratio $\rho(L,H)/\rho(L,0)$ *vs.* L/ξ_H in fig. 3. We are currently trying to test the conclusions and predictions from this calculation on an exactly solvable model and by simulations of random networks of the model of fig. 2*b*).

The results of (16) and (17) mean that, in an infinitely large percolating sample, as $p\to p_c$, $\rho(H)/\rho(0)$ tends to a constant value which depends on H as $H^{0.5}$ (the exponent value 0.5 is not exact—it represents our subjective approximation to the values 0.526 found in (17)). From fig. 3 it is clear that the scaling behaviour dominates already at $H=4$ (*i.e.* when $r_{H0}/r_0 \approx \approx \omega_c\tau \approx 4$). Therefore, it should not be difficult to make real percolating systems where our predictions can be tested experimentally.

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