Estimation of the Josephson critical current of a single grain: percolation model of the resistive transition

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Abstract

The granular HTS is treated here as a S-N mixture. Experimental data are used to determine the percolation threshold \( f_0 \) (the volume fraction of superconducting grains at zero resistance) and \( f_p \) (corresponding to the appearance of the first spanning superconducting cluster). The latter consists of percolating channels, each carrying the Josephson current while the former is described within the effective-medium theory. In a S-N mixture we define the "zero-resistance" condition 

\[ I_{ch} \quad (\text{"ch" means } \text{’channel’}) \text{ without destroying superconductivity in the weak links between grains.} \]

\( I_{ch} \) depends on the direction of the current and on temperature. The maximal superconducting current that the spanning cluster can withstand is 

\[ I_{cl} = nS I_{ch}, \]

where 'cl' means 'cluster' and \( S \) is the cross section of the sample. Obviously \( nS \) is the total number of the channels that cross the sample. Zero resistance is achieved at \( f = f_0 = f(T_0) \) when the spanning cluster carries the total transport current: \( I_{cl}(T_0) = I \). Here \( f_0 \) is a function of \( I \) and \( f_0(I) \geq f_p \), where the equality is reached at very weak transport currents \( I \sim I_{ch} \). The latter condition can be hardly realized experimentally, since \( I_{ch} \approx 0.1–10 \mu A \), as we show below.

Let us find both \( f_p \) and \( f_0 \) for our sample. The exact results obtained using the percolation theory (see Refs. [1–4] as reviews) are \( f_p^{\text{cont}} \approx 0.17 \) for continuum and \( f_p^{\text{latt}} \approx 0.20 \) for dense lattices. These results were recently corroborated experimentally [5]. We accept \( f_p = f_p^{\text{cont}} \approx 0.17 \) for our sample. Since \( f_p = f(T_p) \), we get \( T_p \approx 80 \text{ K} \).

At \( f > f_p \approx 0.17 \) the S-N mixture consists of the parallely connected spanning superconducting cluster and the non-spanning network. The voltage across the spanning cluster is zero for currents smaller than \( I_{cl} \). At higher currents the weak links become normal and the voltage drop rises steeply. The non-spanning part has a smooth V–I characteristic. Therefore, we can assume that the spanning cluster carries a current that slightly exceeds \( I_{cl} \) in order to keep voltages across the two parallel resistors equal. Correspondingly, the non-spanning part carries slightly less than \( I - I_{cl} \). In order to find \( I_{cl} \), let us notice that \( n \propto \eta^{-2} \), where \( \eta \propto (f - f_p)^{-v} \) is the correlation length of the percolation problem and \( v \approx 0.9 \) [1]. Thus we get

\[ n_{ab} = \gamma (f - f_p)^{2v}/s_{ab} \quad (I \parallel ab), \]

\[ n_c = \gamma (f - f_p)^{2v}/s_c \quad (I \parallel c), \]

where \( \gamma \approx 1 \) and \( s_{ab}, s_c \) are the mean cross sections of the grain for \( I \parallel ab \) and \( I \parallel c \), respectively. Note that \( n_{ab}/n_c = m \approx 100 \). Zero resistance is achieved at \( j = nI_{ch} \). Using

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experiment, therefore, which determine the bulk critical current, remains finite, so just below \( T_c \) and \( f_{\text{c}} \). Correspondingly, the condition that determines the current direction. Note that \( j \) becomes "field-sensitive" provided \( S \) density and \( n \) stands for either \( \rho_{\text{c}} \) or \( \rho_{\text{sc}} \), depending on the current direction. This confirms our assertion that \( j \) is constant in our experiment, therefore \( I_{\text{ch}} / I_{\text{c}} \approx m \approx 100 \).

As one can see from Fig. 1, even weak fields \( H \leq 750 \) Oe affect the resistive transition appreciably. The excess resistivity, \( \Delta \rho \), arising due to the magnetic field is apparently almost independent on the direction of the field with respect to the grain orientation or to the direction of the current. This confirms our assertion that the field affects the intergrain point-like weak links and that the resistivity associated with the vortex motion is unimportant in our case.

The resistive transition becomes "field-sensitive" below certain temperature \( T_H \) that can be estimated from Fig. 1 as \( T_H \approx 92 \) K for \( I_{\|ab} \) and \( T_H \approx 75 \) K for \( I_{\|c} \). Above \( T_H \) the resistivity is almost field independent. This can be expected, noticing that the Josephson critical current \( I_{\text{ch}} \) vanishes as \( T \to T_{\text{onset}} \), whereas \( I_g \), the grain bulk critical current, remains finite, so just below \( T_{\text{onset}} \) we have \( I_{\text{ch}} \ll I_g \). In this situation the magnetic field, that further suppresses \( I_{\text{ch}} \), has almost no effect on \( \rho(T) \). The latter becomes "field-sensitive" provided \( I_{\text{ch}} \approx I_g \). Correspondingly, the condition that determines \( T_H \) is

\[
I_{\text{ch}}(T_H) \approx I_{\text{c}}(T_H) \approx I_g.
\]

Since \( I_{\text{ch}} / I_{\text{c}} = m \) is of the order of the ratio of the cross-sectional areas of the grains in the corresponding directions (thus for BSCCO one gets \( m \approx 100 \)), it is clear that \( T_H \) should be considerably less than \( T_{\text{c}} \), as indeed observed experimentally. Moreover, using Eq. (5) one can obtain an estimate of \( I_{\text{ch}} \), specifically \( I_{\text{ch}}(92 \text{ K}) \approx 0.03 \mu A \) and \( I_{\text{ch}}(75 \text{ K}) \approx 2 \mu A \). The fact that \( T_H^0 > T_p \), i.e., the effect of the magnetic field on \( \rho_{\text{ab}} \) becomes noticeable even in the absence of the spanning cluster, does not contradict our approach. Suppression of \( I_{\text{ch}} \) results in appearance of additional resistivity in the non-spanning part also, but the quantitative analysis of this effect is quite complex and will be carried out elsewhere.

There are two problems regarding the applicability of the effective medium approximation (EMA) in these ceramics. First, the presence of finite \( I_{\text{ch}} \) results in additional resistivity not accounted for by EMA. Second, the percolation of current via the spanning cluster breaks the consistency of the EMA and requires consideration of the two-resistor model. Thus under an applied magnetic field of \( H = 750 \) Oe, we get two competing effects: the appearance of additional resistivity due to suppression of weak links that should worsen the EMA fit, and a suppression of \( I_{\text{ch}} \) that works in favor of applicability of the EMA. The latter effect proves to be decisive, since the spanning cluster is responsible for the very fast drop of \( \rho_{\text{ab}} \) at \( T < T_{\text{ch}} \) in the absence of the field. Therefore it is not surprising that at \( H = 750 \) Oe we get a consistent EMA picture at all \( T > T_{\text{ch}} \approx 75 \) K, whereas at \( H = 0 \) the applicability of the EMA is restricted by the condition \( T > T_p \approx 80 \) K.

Acknowledgements

This research was supported by the ISF grant 559/98 and by the BSF grant 98-370. E. M. and Y. M. S. acknowledge support of the KAMEA Fellowship program.

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